8/11/13 QUESITI

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① $7+1 = 8-2 = 6$

$13-6 = (7)$

② $\mathbb{R}^5 \longrightarrow \mathbb{R}^7$

$5-1 = 4 \qquad 1 < \dim \mathrm{Im}(f) < 4$

deve arrivare a 5

③ $B = \left\{\begin{pmatrix}1\\2\end{pmatrix}, \begin{pmatrix}3\\1\end{pmatrix}\right\} \qquad C = \left\{\begin{pmatrix}5\\-1\end{pmatrix}, \begin{pmatrix}-2\\3\end{pmatrix}\right\} \qquad [f]_B^C = \begin{pmatrix}4 & -1\\ 3 & 2\end{pmatrix} \qquad f\begin{pmatrix}2\\-1\end{pmatrix}$?

1ª colonna di $[f]_B^C$ e la moltiplico per C, 2ª colonna con C

$4\begin{pmatrix}5\\-1\end{pmatrix} + 3\begin{pmatrix}-2\\3\end{pmatrix} = \begin{pmatrix}20 & -6\\ -4 & +9\end{pmatrix} = \begin{pmatrix}14\\5\end{pmatrix}$

$-1\begin{pmatrix}5\\-1\end{pmatrix} + 2\begin{pmatrix}-2\\3\end{pmatrix} = \begin{pmatrix}-5 & -4\\ -1 & 6\end{pmatrix} = \begin{pmatrix}-9\\7\end{pmatrix}$

$[f]_B = \begin{pmatrix}14\\5\end{pmatrix}\begin{pmatrix}-9\\7\end{pmatrix}$

$B = \left\{\begin{pmatrix}1\\2\end{pmatrix}, \begin{pmatrix}3\\1\end{pmatrix}\right\}$

$\begin{cases} a+2b = 14 \\ c+2d = 5 \\ 3a+b = -9 \\ 3c+d = 7 \end{cases}$

$a = -2b+14 \Rightarrow a = -2\cdot\left(\frac{51}{5}\right)+14 = -\frac{102}{5}+14 = \frac{-102+70}{5} = \left(-\frac{32}{5}\right)$

$c = -2d+5 \Rightarrow c = -2\left(\frac{8}{5}\right)+5 \Rightarrow -\frac{16}{5}+5 = \frac{-16+25}{5} = \left(\frac{+9}{5}\right) \qquad b = \left(\frac{51}{5}\right)$

$3(-2b+14)+b+9 = 0 \Rightarrow -6b+42+b+9 = 0 \Rightarrow -5b+51 = 0 \quad 5b-51 = 0$

$3(-2d+5)+d-7 = 0 \Rightarrow -6d+15+d-7 = 0 \Rightarrow -5d+8 = 0 \quad -5d-8 = 0 \Rightarrow d = \left(\frac{8}{5}\right)$

$\begin{pmatrix}a & b\\ c & d\end{pmatrix} = \begin{pmatrix}-\frac{32}{5} & \frac{51}{5}\\ -\frac{9}{5} & \frac{8}{5}\end{pmatrix}\begin{pmatrix}2\\-1\end{pmatrix} = \begin{pmatrix}-\frac{64}{5} - \frac{51}{5}\\ \frac{18}{5} - \frac{8}{5}\end{pmatrix} = \frac{64-51}{5} = \frac{-115}{5} = -23\,\ell_1$

$\frac{18-8}{5} = \frac{10}{5} = 2\,\ell_2$

④ $\begin{pmatrix}1 & 3 & -1\\ 4 & -5 & 1\\ -2 & 1 & 3\end{pmatrix} \qquad (A^t)_{12}$?

$-15-6-4-(-10+1+36) = -25+10-1-36 = -52$

$\begin{pmatrix}1 & 4 & -2\\ 3 & -5 & 1\\ -1 & 1 & 3\end{pmatrix} = 9-(-1) = 9+1 = -10$

$\frac{-10}{-52} = \frac{5}{26}$ sbagliata $= \frac{5}{26}$

(6) $z^2 - (5+i)z + (1-i) = 0$

$\Delta = (3+i)^2 - 4(1-i) = 9 - 1 + 6i - 4 + 4i = 4 + 10i$

$z_{1,2} = \frac{5+i \pm \sqrt{30i-16}}{2}$

$a^2 - b^2 = -16$ $\quad a = -3$

$2abi = 30i \Rightarrow ab = 15$ $\quad b = -5(i)$

(7) $\{x \in \mathbb{R}^3 : 2x_1 + 3x_2 - 5x_3 = 0\}$ $\quad y = \begin{pmatrix} 4 \\ -1 \\ 2 \end{pmatrix}$ $\quad$ Дадена $\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$ $\quad \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = 2 + 3 - 5 = 0$

$\begin{pmatrix} 3 \\ -2 \\ 0 \end{pmatrix} = 6 - 6 + 0 = 0$

$\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = a\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + b\begin{pmatrix} 3 \\ -2 \\ 0 \end{pmatrix} + c\begin{pmatrix} 4 \\ -1 \\ 2 \end{pmatrix}$

$\begin{cases} a + 3b + 4c = 1 \\ a - 2b - c = 1 \\ a + 2c = -1 \end{cases}$ $\quad \begin{cases} 3b = -a - 4c + 1 \\ -2c - 1 - 2b - c + 1 = 0 \\ a = -2c + 1 \end{cases}$ $\quad 3b = 2c + 1 - \frac{4}{3}\left(-\frac{2b-2}{3}\right) + 1 \Rightarrow$ $\quad 3b = 2c + 1 + \frac{8b}{3} + \frac{8}{3} + 1$

$\Rightarrow -3c - 2b - 2 = 0$ $\quad 3c + 2b + 2 = 0$ $\quad c = -\frac{2b-2}{3}$

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1) $8+1 = 9-2 = 7-4 = 3$

2) $\mathbb{R}^4 \to \mathbb{R}^8$

$4-1 = 3$ $\quad 1 \le \dim \le 3$

3) $B = \left\{\begin{pmatrix}2\\1\end{pmatrix}, \begin{pmatrix}1\\3\end{pmatrix}\right\}$ $\quad C = \left\{\begin{pmatrix}4\\-1\end{pmatrix}, \begin{pmatrix}-3\\2\end{pmatrix}\right\}$ $\quad [f]_B^C = \begin{pmatrix}3 & -1\\2 & 5\end{pmatrix}$ $\quad f\begin{pmatrix}1\\-2\end{pmatrix}?$

$$3\begin{pmatrix}4\\-1\end{pmatrix} + 2\begin{pmatrix}-3\\2\end{pmatrix} = \begin{pmatrix}12-6\\-3+4\end{pmatrix} = \begin{pmatrix}6\\1\end{pmatrix}$$

$$-1\begin{pmatrix}4\\-1\end{pmatrix} + 5\begin{pmatrix}-3\\2\end{pmatrix} = \begin{pmatrix}-4-15\\1+10\end{pmatrix} = \begin{pmatrix}-19\\11\end{pmatrix}$$

$$[f]_B = \begin{pmatrix}6 & -19\\1 & 11\end{pmatrix}$$

$$B = \begin{pmatrix}2 & 1\\1 & 3\end{pmatrix}$$

$$\begin{cases} 3a+b=6 \\ 2c+d=1 \\ a+3b=-19 \\ c+3d=11 \end{cases}$$

$2(-3b-19)+b-6=0 \Rightarrow -6b-38+b-6=0 \Rightarrow -5b-44=0 \quad b=-\frac{44}{5}$

$7(-3d+11)+d-1=0 \quad -6d+22+d-1=0 \quad -5d+21=0 \quad d=\frac{21}{5}$

$a=-3b-19 \Rightarrow a=-3\left(-\frac{44}{5}\right)-19 = \frac{132}{5}-19 = \frac{132-95}{5} = \frac{37}{5}$

$c=-3d+11 \Rightarrow c=-3\left(\frac{21}{5}\right)+11 = -\frac{63}{5}+11 = \frac{-63+55}{5} = -\frac{8}{5}$

$$\begin{pmatrix}a & b\\c & d\end{pmatrix} = \begin{pmatrix}\frac{37}{5} & -\frac{44}{5}\\ -\frac{8}{5} & \frac{21}{5}\end{pmatrix} \cdot \begin{pmatrix}1\\-2\end{pmatrix} = \begin{pmatrix}\frac{37}{5}+\frac{88}{5}\\ -\frac{8}{5}-\frac{42}{5}\end{pmatrix} \quad \frac{37+88}{5} = \frac{125}{5} = 25 \ell_1$$

$$\frac{-8-44}{5} = -\frac{50}{5} = -10 \ell_2$$

4) $\begin{pmatrix}-2 & 3 & -1\\5 & -2 & 1\\4 & 1 & 3\end{pmatrix} \quad (A^{-1})_{32}$

$12+12-6-(8-2+45) = 24-5-8+2-45 = -32$

$$\begin{pmatrix}-2 & 4 & 4\\3 & -1 & 1\\-1 & 1 & 3\end{pmatrix} = -2-(12) = +2-14 = +14$$

$$\frac{14}{32} = -\frac{7}{16}$$

6) $6z^2 - \underbrace{(2+i)}_{b} z + \underbrace{1+i}_{c}$ $\quad -4(6+6i) \quad -24+24i$

$a^2-b^2 = -21$

$\Delta = (2+i)^2 - 24 - 24i = 4-1+4i-24-24i = -21-20i \quad 2abi = -20i \Rightarrow ab = -10$

$z_{1,2} = \frac{2+i \pm \sqrt{-21-20i}}{12} \quad \frac{2+i \pm (2-5i)}{12}$

$\frac{4-4i}{12} - \frac{1}{3} - \frac{1}{3}i$

$\cdot 0 \quad -5i \quad \frac{6i}{12} - \frac{1}{2}$

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(1) $1-i=7$ $5-i=3$

(2) $B=\begin{pmatrix} 3 & 1 \\ -1 & 4 \end{pmatrix}$ $\begin{bmatrix} 5 \\ -7 \end{bmatrix}_B$ $\begin{bmatrix} 3e_1-e_2 \\ -e_1+4e_2 \end{bmatrix}_B$

$\begin{pmatrix} 5 \\ -7 \end{pmatrix} = x\begin{pmatrix} 3 \\ -1 \end{pmatrix} + y\begin{pmatrix} 1 \\ 4 \end{pmatrix}$ $B\begin{bmatrix} 5 \\ -7 \end{bmatrix}_B$ $\begin{cases} 3x+y=5 \\ -x+4y=-7 \end{cases}$ $\begin{cases} 5x+4(2-3)-5=0 \\ x=2-3=-1 \end{cases}$ $5x+8-12-5=0$

$\begin{pmatrix} 2 \\ -1 \end{pmatrix}$

(3) $X=5$

$Y=3$ $X+Y=5-1=7$

$5<\dim X+Y<7$

(4) $\begin{cases} 5x+2y-5z=4 \\ 3x-4y+2z=13 \\ x+3y-3z=-5 \end{cases}$ $\left(\begin{array}{ccc|c} 5 & 2 & -5 & 4 \\ 3 & -4 & 2 & 13 \\ 1 & 3 & -3 & -5 \end{array}\right)$ $\left(\begin{array}{ccc|c} 5 & 2 & -5 & 4 \\ 3 & -4 & 2 & 13 \\ 2 & 6 & -5 & -9 \end{array}\right)$ $\left(\begin{array}{ccc|c} 5 & 2 & -5 & 4 \\ 0 & -6 & 5 & 19 \\ 0 & 0 & 0 & 10 \end{array}\right)$

$\begin{cases} 5x+2y-3z=4 \\ -3x-6y+5z=9 \end{cases}$ $\begin{cases} \\ 2x+5y-5z=-9 \end{cases}$ $z=t$ $\begin{cases} 5\left(\frac{-6y+5t-9}{2}\right)+2y-3z=4 \\ x=\frac{-6y+5t-9}{2} \end{cases}$ $\begin{cases} \frac{-30y}{2}+\frac{25t}{2}-\frac{45}{2}+2y-3t=4 \\ \end{cases}$

$-13y-53+19t=0 \Rightarrow 13y+\frac{53}{2}-\frac{19}{2}t \Rightarrow y=\frac{-53-19t}{26}$

$x=\frac{-6\left(\frac{-53-19t}{26}\right)+5t-9}{2} \Rightarrow \frac{\frac{318}{26}+\frac{114t}{26}+5t-9}{2} \Rightarrow \frac{\frac{318-234}{26}+\frac{114+130t}{26}}{2}$

$\frac{84}{26}\cdot\frac{1}{2}+\frac{244}{26}t\cdot\frac{1}{2}$

(5) $\det\begin{pmatrix} 1-i & 2+i \\ i-2 & 4+2i \end{pmatrix}=0$

$(4-i)(1+2i)-(i-2)(2+i)=4+2i-i-2i^2-(2i+i^2-2i-2i)=4+2i-i+2-2i+i+2i+3=$

$(-\lambda-3i)^2=\lambda+3 \Rightarrow \lambda^2+3+(3+2i)\lambda+3i=0 \Rightarrow \lambda^2+\lambda(3+2i)+(3+3i)=0$

$\frac{2(2+i)}{(2+i)}-\frac{-4+(-2-i)}{(2+i)(2-i)}=-\frac{-3+4i+2i-i^2}{4-2i+2i-i^2}=\frac{-3+6i+1}{4+1}=\frac{-2+6i}{5}=\frac{1}{5}(-2+6i)$

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6) $\begin{pmatrix} 3 & 2 & -1 & 1 \\ 1 & -4 & -5 & -1 \\ 4 & 5 & 3 & 2 \end{pmatrix}$ $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ $\begin{pmatrix} 2 & 4 \\ 5 & 2 \end{pmatrix}$

$\begin{pmatrix} 2 & -1 & 4 \\ -4 & -5 & -1 \\ 5 & 3 & 2 \end{pmatrix}$ $\begin{pmatrix} 3 & 2 & 4 \\ 1 & -4 & -1 \\ 4 & 5 & 2 \end{pmatrix}$

$2 \cdot (-5) \cdot 2 + (-1) \cdot (-1) \cdot 5 + 4 \cdot (-4) \cdot 3 - 5 \cdot (-5) \cdot 4 - 3 \cdot (-1) \cdot 2 - 2 \cdot (-4) \cdot (-1)$

$-20 + 5 - 48 - (-100 - 6 + 8)$

$-63 - (-98) =$

$-63 + 98 = 35$

$3 \cdot (-4) \cdot 2 + 2 \cdot (-1) \cdot 4 + 4 \cdot 1 \cdot 5 - 4 \cdot (-4) \cdot 4 - 5 \cdot (-1) \cdot 3 - 2 \cdot 1 \cdot 2$

$-24 + 8 + 20 - (+64 - 15 + 4)$

$-24 + 8 + 20 + 64 + 15 - 4$

$-36 + 99 = 63$

1) $8 = 2 = 6 = 2 = 4$

2) $B = \left(\begin{pmatrix} 3 \\ 1 \end{pmatrix} \begin{pmatrix} -2 \\ 1 \end{pmatrix} \right)$ $\left[\begin{pmatrix} 12 \\ -1 \end{pmatrix} \right]_B$

$\begin{pmatrix} 12 \\ -1 \end{pmatrix} = \alpha \begin{pmatrix} 3 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} -2 \\ 1 \end{pmatrix}$ $\begin{cases} 3\alpha - 2\beta = 12 \\ \alpha + \beta = -1 \end{cases}$ $\begin{cases} 3(-\beta - 1) - 2\beta - 12 = 0 \\ \alpha = -\beta - 1 \end{cases}$ $\begin{cases} -3\beta - 3 - 2\beta - 12 = 0 \\ \to \end{cases}$

$-5\beta - 15 = 0$ $\begin{cases} 5\beta + 15 = 0 \\ \alpha = -(-3) - 1 \end{cases}$ $\beta = -3$ $\Rightarrow +3 - 1 = 2$ $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$

3) $W = 4$

$z = 6$

$W + Z = 10 - 1 = 9$

$6 \leq \dim(W+Z) \leq 9$

4) $\begin{cases} 4x + 2y + 3z = 13 \\ 8x - 8y + 7z = -5 \\ 2x - 3y + 5z = 4 \end{cases}$

$\begin{pmatrix} 4 & 2 & 3 & 13 \\ 8 & -8 & 7 & -5 \\ 2 & -3 & 5 & 4 \end{pmatrix}$ $\begin{matrix} +4 & -2 & -3 & -13 \end{matrix}$ $\begin{pmatrix} -4 & 2 & 3 & 13 \\ 4 & -6 & 10 & 8 \\ 2 & -3 & 5 & 4 \end{pmatrix}$

5) $\det \begin{pmatrix} 2-i & 1+2i \\ 1-i & 1+4i \end{pmatrix} = 0$

$(2-i)(1+4i) - (1-i)(1+i) = 0$ $2 + 8i - i - 4i^2 - (1 + i^2 + i - i) = 0$

$2 + 8i - i - 4i^2 - i - i^2 + i + i = 0$ $-5i^2 + 6i + 2 + i + i = 0$ $5 + 6i + 2 + i + i = 0$

$7 + 6i + i + i = 0$ $z\frac{(1+i)}{(1+i)} = \frac{-7 - 6i}{(1+i)(1-i)} (1-i) = \frac{-7 + 7i - 6i + 6i^2}{2} = \frac{-7 + i - 6}{2} = \frac{1}{2}(-13 + i)$